B.Tech. III sem.
Fluid Mechanics

Sec. A. 1 (i) c (ii) c (iii) c (iv) c (v) c (vi) a (vii) a
(viii) a (ix) · (x) c.

Sec. B Q. 2 (i) Eddy Viscosity :- eddy viscosity E_v is analogus to μ, the absolute viscosity. The relation ship betn. shear stress and velocity gradient in a turbulent stream is used to define an eddy viscosity.

$$\tau_{t}g_c = E_v \frac{du}{dy}$$

(ii) Boundary layer :- Boundary layer is defined as that part of a moving fluid in which the fluid motion is influenced by the presense of a solid boundary.
Short explaination of boundary layer formatn. may be given.

(iii) Transition length :- The length of the entrance region of the tube necessary for the boundary layer to reach the center of the tube and for fully developed flow to be established is called the transition length.

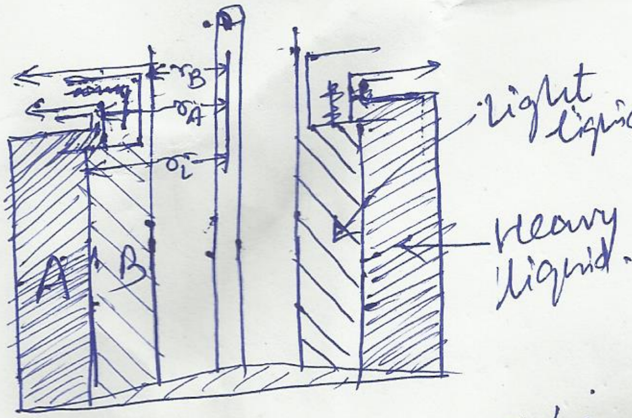
$$\frac{x_t}{D} = 0.05\, N_{Re} \quad \text{(for laminar flow)}$$

(iv) Hydrostatic law :- In a stationary mass of a single static fluid, the pressure is constant in any cross section ‖ to the earth's surface but varies from height to height.

$P = \rho g z$ is the mathematical expression (2)

or

(a) Centrifugal decanter :- It is used when the density difference of the two liquids is so small.



$$P_i - P_B = P_i - P_A$$

$$P_i - P_B = \frac{\omega^2 \rho_B (r_i^2 - r_B^2)}{2 g_c}$$

$$P_i - P_A = \frac{\omega^2 \rho_A (r_i^2 - r_A^2)}{2 g_c}$$

$$\therefore \rho_B (r_i^2 - r_B^2) = \rho_A (r_i^2 - r_A^2)$$

$$\therefore r_i = \sqrt{\frac{r_A^2 - (\rho_B/\rho_A) r_B^2}{1 - \rho_B/\rho_A}}.$$

(b) $\omega = \pi \times \frac{4000}{60} = 209.4$ rad/s.

$\rho = 1109$ kg/m³.

$r_2 = 0.125$ m ; $r_1 = 0.125 - 0.05 = 0.075$.

$$P_2 - P_1 = \frac{\omega^2 \rho (r_2^2 - r_1^2)}{2 g_c}$$

$$= \frac{(209.4)^2 \times 1109 \times [(0.125)^2 - (0.075)^2]}{2}$$

$= 243139$ N/m².

0.015625 − 0.005625

0.010000

Q. 3. $A_1 V_1 = A_2 V_2$ $V_2 = 20\ m/s$, $Q = 10\ m^3/s$. (3)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}.$$

$$\frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{(10)^2}{2 \times 9.81} = \frac{P_2}{\rho g} + \frac{(20)^2}{2 \times 9.81}.$$

$$P_2 = 29.25\ KN/m^2$$

$$F_x = \rho Q [V_{1x} - V_{2x}] + (P_1 A_1)_x + (P_2 A_2)_x.$$

$(P_1 A_1)_x = P_1 A_1 = 29.43\ KN/m^2 \times 1 = 29.43\ KN.$

$(P_2 A_2)_x = -P_2 A_2 \cos 45 = -29.25\ \frac{KN}{m^2} \times 0.5 \times 0.7071$

$$F_x = 19038.59\ N.$$

$$F_y = \rho Q [V_{1y} - V_{2y}] + (P_1 A_1)_y + (P_2 A_2)_y.$$

$V_{1y} = 0$, $V_{2y} = V_2 \sin 45 = 20 \times 0.7071 = 14.142$

$(P_1 A_1)_y = 0$, $(P_2 A_2)_y = -P_2 A_2 \sin 45.$

$= -29255.8 \times 0.5 \times \sin 45$

$= -10343.37$

$$F_y = -10507.42\ N.$$

Resultant force $F_R = \sqrt{F_x^2 + F_y^2} = \underline{21746.6\ N}$

Or.

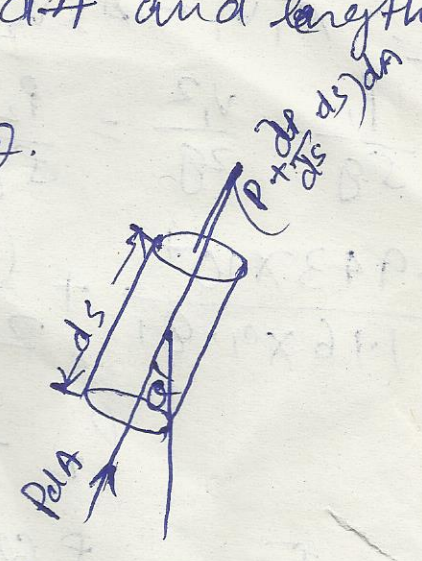
<u>Bernoulli's theorem</u> :- Euler's eqn. of motion -

consider a stream line in which flow is taking place in S-direction. Consider a cylindrical element of C.S. dA and length ds (4)

The force balance -

$PdA - (P + \frac{\partial P}{\partial s} ds)dA - \rho g\, dA\, ds \cos\theta$

$= \rho dA ds\, a_s$.



a_s = acceleraⁿ in s direcⁿ.

$a_s = \frac{dv}{dt}$

$= \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v}{\partial t} \cdot \frac{dt}{dt} = \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t}$

if the flow is steady, $\frac{\partial v}{\partial t} = 0$.

$a_s = \frac{v \partial v}{\partial s}$,

$\therefore -\frac{\partial P}{\partial s} \cdot ds dA - \rho g\, dA ds \cos\theta = \rho dA ds \times \frac{v \partial v}{\partial s}$

Dividing by $\rho ds dA$; $-\frac{\partial P}{\rho \partial s} - g\cos\theta = \frac{v \partial v}{\partial s}$

Or $\frac{\partial P}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$.

But $\cos\theta = \frac{dz}{ds}$.

$\therefore \frac{1}{\rho}\frac{\partial P}{\partial s} + g\frac{dz}{ds} + \frac{v \partial v}{\partial s} = 0$ Or $\frac{\partial P}{\rho} + g dz + v dv = 0$

Or $\frac{dP}{\rho} + g dz + v dv = 0$

Bernoulli's equⁿ is obtained by integrating the Euler's equⁿ of motion as

$\int \frac{dP}{\rho} + \int g dz + \int v dv = \text{Const}$.

if flow is incompressible, ρ is const. and,

$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{Cost}$.

Or $\boxed{P/\rho g + z + v^2/2g = C}$ ← Bernoulli's equⁿ.

Q. 1. Hagen Poiseuille equ^n. :- $\Delta P_s = \frac{32 \Delta L \bar{V} \mu}{g_c D^2}$ (5)

For laminar flow of newtonian fluids in circular pipe, the local velo. u depends only on the radius r. Also, the element of area ds is that of a thin ring of radius r with width dr. The area of this elementry ring is

$$dS = 2\pi r \, dr.$$

The desired velocity distribution relation is u as a function of r.

A direct method of obtaining the velo. distribution for newtonian fluids is to use the definition of viscosity.

$$\mu = -\frac{\tau g_c}{du/dr} \quad (1)$$

$$\quad (2)$$

as we know, $\frac{\tau_w}{r_w} = \frac{\tau}{r}$

$$\therefore \frac{du}{dr} = -\frac{\tau g_c}{\mu} = -\frac{\tau_w g_c}{r_w \mu} \cdot r \quad (3)$$

integra^n. of (3) with the B.C. $u=0$, $r=r_w$; gives

$$\int_0^u du = -\frac{\tau_w g_c}{r_w \mu}\int_{r_w}^{r} r \, dr$$

$$u = \frac{\tau_w g_c}{2 r_w \mu}(r_w^2 - r^2)$$

$$u_{max} = \frac{\tau_w g_c r_w}{2\mu} \quad [\because r=0]$$

~~Thus~~ r_w for average velo. $\bar{V}$, we use, $ds = 2\pi r\,dr$

$$u = \frac{\tau_w g_c}{2 r_w \mu}\left(r_w^2 - r^2\right)$$

$$S = \pi r_w^2$$

$$\bar{V} = \frac{1}{S}\int_S u\,ds$$

$$= \frac{1}{\pi r_w^2}\int_S \frac{\tau_w g_c}{\cancel{2}\cancel{\pi} r_w \mu}\left(r_w^2 - r^2\right)\cancel{2\pi} r\,dr$$

$$= \frac{\tau_w g_c}{r_w^3\,\mu}\int_0^{r_w}\left(r_w^2 - r^2\right) r\,dr$$

$$\bar{V} = \frac{\tau_w g_c r_w}{4\mu} \quad \text{— (4)}$$

Now, we transform this eqn. (4) by eleminating r_w in favour of ΔP_s by using eqn.

$$h_{fs} = \frac{2\tau_w}{\rho r_w}\Delta L = \frac{\Delta P_s}{\rho} = 4f\frac{\Delta L \bar{V}^2}{2 D g_c}$$

$$\bar{V} = \frac{\Delta P_s g_c}{\Delta L}\frac{r_w}{2}\frac{r_w}{4\mu} = \frac{\Delta P_s g_c D^2}{32 \Delta L \mu}$$

$$\therefore \Delta P_s = \frac{32 \Delta L \bar{V} \mu}{g_c D^2}$$

Limitations :- applicable only for laminar flow, Newtonian fluid, circular channel.

Significance - Used for practical calculaⁿ. of viscosity.

(a) or

$$\mu = -\frac{\tau g_c}{du/dr}, \quad \frac{\tau_w}{r_w} = \frac{\tau}{r}$$

$$\frac{du}{dr} = -\frac{\tau g_c}{\mu} = -\frac{\tau_w g_c}{r_w \mu}\cdot r$$

$$\int_0^u du = -\frac{\tau_w g_c}{r_w \mu}\cdot\int_{r_w}^{r} r\,dr$$

$$u = \frac{\tau_w g_c}{2 r_w \mu}(r_w^2 - r^2)$$

$$u_{max} = \frac{\tau_w g_c r_w}{2\mu} \quad [r=0]$$

for average velo. $\bar{V}$,

$$ds = 2\pi r\,dr, \quad u = \frac{\tau_w g_c}{2 r_w \mu}(r_w^2 - r^2)$$

$$s = \pi r_w^2, \quad \bar{V} = \frac{1}{S}\int_s u\,ds.$$

$$\bar{V} = \frac{1}{\pi r_w^2}\int_s \frac{\tau_w g_c}{2 r_w \mu}(r_w^2 - r^2)\,2\pi r\,dr$$

$\cancel{\bar{V} = \tau_w}$

$$\frac{u}{u_{max}} = 1 - (r/r_w)^2.$$

(b) Fanning friction factor - which is a common parameter especially useful in the study of turbulent flow. It is denoted by f. and may be defined as the ratio of the wall shear stress to the product of the density and the velo. head

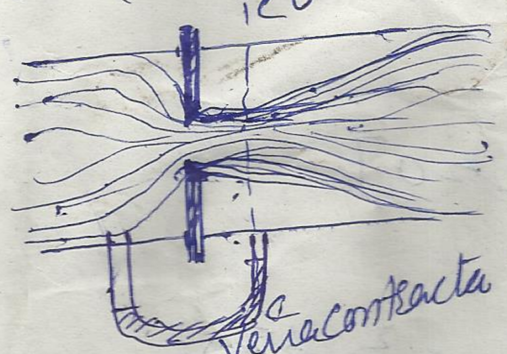
$$\therefore f = \frac{\tau_w}{\rho \bar{V}^2/2g_c} = \frac{2 g_c \tau_w}{\rho \bar{V}^2}$$

$$h_{fs} = \frac{2\tau_w}{\rho r_w}\Delta L = \frac{\Delta P_s}{\rho} = 4f\frac{\Delta L}{D}\frac{\bar{V}^2}{2g_c}$$

$$f = \frac{\Delta P_s g_c D}{2\Delta L \rho \bar{V}^2}$$

Unit IV

(5) (a) Orifice meter - It is a variable head flow meter. It meets the objections to the venturi but at the price of a larger power consumption.



It consists of an accurately machined & drilled plate mounted betn. two flanges with the hole concentric with the pipe in which it is mounted. Press. taps, one above and one below the orifice plate are installed and connected to a manometer. working principle:- The reduction of the cross section of the flowing stream in passing through the orifice increases the velocity head at the expense of the press. head and the reduction in pressure between the taps is measured by the manometer. Bernoulli's eqn. provides a basis for correlating the increase in velocity head with the decrease in press. head.

Because of the sharpness of the orifice, the fluid stream separates from the downstream side of the orifice plate and forms a free flowing jet in the downstream ~~fluid~~ side fluid; a vena contracta forms. The jet is not under the control of solid walls, and the area of the jet varries from that of the opening to that of the vena contracta. The area at the vena-contracta is minimum.

$$u_0 = \frac{C_o}{\sqrt{1-\beta^4}} \cdot \sqrt{\frac{2g_c(P_a-P_b)}{\rho}}.$$

(b) The press. drop in the upstream cone is utilized to measure the rate of flow and the decreased velocity is largely recovered in the downstream cone. To make the press. recovery large, the angle of the downstream cone is small so boundary layer separation is prevented and friction minimized. Since separation does not occur in a contracting cross section, the upstream cone can be made shorter than the downstream cone.

Or

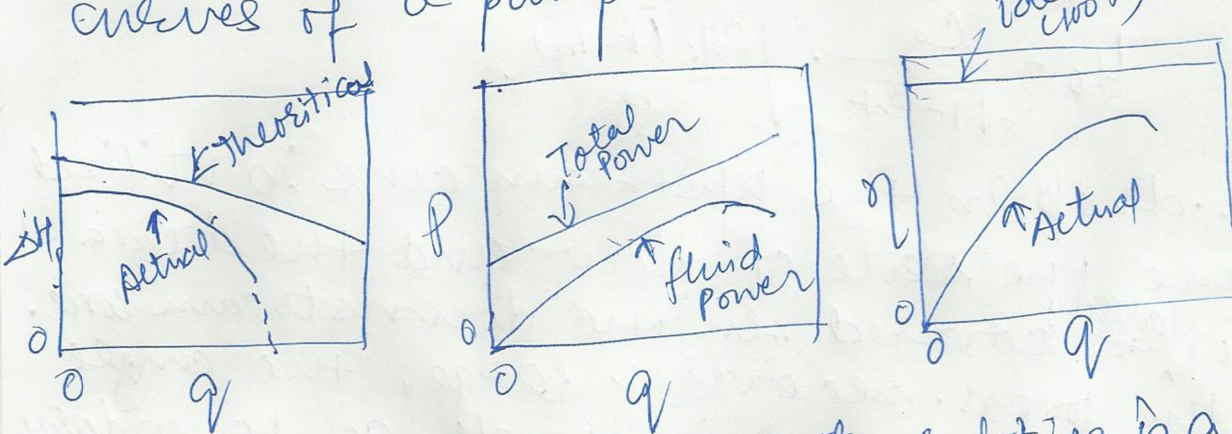
(i) NPSH and cavitation:- If the suction press. is less or slightly higher than the vapor press., some liquid may flash to vapor inside the pump, this is called cavitation

To avoid cavitation, the pressure at the pump inlet must exceed the vapor press. by a certain value, called the net positive suction head (NPSH). The required value of NPSH is about 2 to 3 m for small centrifugal pumps

$$NPSH = \frac{g_c}{g}\left(\frac{P_a' - P_v}{\rho} - h_{fs}\right) - Z_a.$$

P_a' = absolutte press. at surface of reservoir
P_v = vap. press., h_{fs} = friction in suction line.

(ii) Characteristics curves:- The plots of actual head, total power consumption, and efficiency v/s vol. flow rate are called the characteristics curves of a pump.



The theoritical head-flow rate relation is a straight line, the actual developed head is considerably less and drops to zero as the rate increases to a certain value in a given pump.

~~The difference between~~ Typical curves of fluid power and total power v/s flow rate are shown. The difference between ideal & actual performance represents the power lost in the pump it results from fluid friction and shock losses.

The pump efficiency is the ratio of fluid power to the total power input. The curve shows that the efficiency rises rapidly with flow rates at low rates, reaches a maximum in the region of the rated capacity, then falls as the flow rate approaches the zero-head value.

~~Unit-V~~ Unit V
OR

~~6-(a)~~ $P = \dfrac{N_p n^3 D_a^5 \rho}{g_c}$ $N_p = 5.8$, $n = \dfrac{90}{60} = 1.5\ r/s$

$P = 5.8 \times (1.5)^3 \times (0.5)^5 \times 93.5$ = 2476.6 W = 2.476 kW

Power, $\frac{1821}{550}$ = 3.31 hp (2.47 kW) or $5.8 \times (1.5)^3 \times (0.5)^5 \times 93.5$